War as an Investment

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Abstract

States often bargain over objects that affect their future bargaining power. A large territory, for example, is not only valuable in itself, but also as a source of raw material, population and defense. As a result, states not only try to maximize their benefits when they negotiate over the partition of a territory; they also strive to increase their power—their ability to secure a favorable outcome in the future. We study these situations in the context of two- and three-player bargaining games in which present outcomes affect future power, and show three main results: (i) in a two-player negotiation, war never occurs in equilibrium, unless states value the future too differently; (ii) with three players, war can occur in equilibrium; (iii) however, these wars of investments—wars aimed at accumulating resources that improve their ability to secure favorable outcomes in future negotiations—only occur if there are increasing returns in the mapping from resources to the probability of winning.

The defining feature of international relations is the absence of an overarching government that regulates the relations between states. This state of 'anarchy' implies that countries must rely on their own means to secure the outcomes they value most. For the most part, their shares of the pie are not determined by laws or supranational institutions, but by their relative power—their ability to impose costs and offer benefits. Powerful countries tend to impose their views and to obtain a larger share of the pie, whereas weak ones often struggle to survive.

In their interactions, states must therefore not only concern themselves with the share of benefits they obtain, but also with how this share affects their ability to secure favorable outcomes in future negotiations—in short, their power. Indeed states often, if not always, negotiate over object that affect their future power. A larger territory, for example, is not only valuable in itself, but is also a source of raw materials and population—two important sources of power. Present gains affect future bargaining power, and whether negotiations break down—whether war occurs or not—must be analyzed taking these strategic considerations into account.

Here we ask when states fight wars of investment—wars aimed at accumulating resources that improve their ability to secure favorable outcomes in future negotiations. Two bargaining games with complete information are analyzed. First, we show that with two players only, war never occurs in equilibrium. This result was first derived by Fearon (1996), but his assumption that war terminates the game ruled out the possibility of wars of investment. We therefore extend the model to account for continuous war gains, and show that Fearon's result holds regardless. However, we also find that war can occur if states value the future too differently.

We then analyze a three-player game and show that here, war can occur in equilibrium, even under complete information. However, we also show the surprising result that war occurs if, and only if, there are increasing returns in the mapping from capabilities to power. This result contrasts with two main bodies of literature. It first demonstrates that two states can plausibly have a positive expected utility for war, even under complete information and without risk-acceptance—in contrast to Fearon (1995); second, it challenges the classical balance of power and realist theories by showing that cumulativity, even when it is large, is not sufficient to make war profitable (Morgenthau 1948, Mearsheimer 2001).

The paper proceeds in three steps. First comes a discussion of existing approaches to the causes of war and an illustrative example that existing models would have difficulty accounting for. Second, a simple extension of Fearon (1996) is proposed, in which the outcome of war is a new partition of the territory instead of terminating the game. Third, a simple three-player model shows that war can occur in equilibrium under complete information. Finally, I identify factors that can affect the incentive to fight wars of investment.

1 Rationality of War

Over the past twenty years, scholars have attempted to explain the causes of war rigorously by analyzing a wide variety of bargaining models and deriving the conditions under which rational actors may fight (Morrow 1989, Fearon 1995, Powell 1999, 2004, 2006).[1] Three central mechanisms have been the focus of attention. First, private information creates incentives to misrepresent, such that both parties in a dyad can have a positive expected utility for fighting (Jervis 1976, Fearon 1995); second, war can occur when large shifts in relative power are so rapid that the declining state prefers fighting now than waiting until tomorrow, when he will be weaker (Powell 2006, Kugler & Organski 1989, Morrow & Kim 1992). Finally, indivisibities can make it impossible for the parties to locate a self-enforcing agreement (Walter 2000, Toft 2003, Goddard 2006).[2]

Unfortunately, these models provide at best an incomplete explanation for wars of investment—wars in which states fight over stakes whose intrinsic worth might seem small when compared to the costs of fighting, but that states value as instruments of future power.

1.1 Wars of Investment

Consider for example the Eighty Years' War (a.k.a. 'Dutch revolt'), in which Spain fought the Seventeen Provinces of the Netherlands (1568–1648). At the origin of the conflict lay tensions accumulated during Philip II's reign over heavy taxation, the repression of Protestantism and centralization efforts, which ultimately led to rioting and armed revolt in the Netherlands. The war lasted longer than any other uprising in modern European history and was ruinous.[3]

From the point of view of existing rationalist explanations for war, this is puzzling. First, misperceptions or asymmetric information can only provide an odd reading of the conflict, especially given its intensity and duration. Indeed, as early as 1574, it was clear to Philip II that "it is a terrible situation and it is getting worse every day" and that "[to speak of] conquering [the Netherlands] by force is to speak of a war without end."[4] Rapid changes in relative power were never a source of concern either, and there was nothing indivisible about the rebels demands.[5] Why, then, did Spain undertake this war and, even more puzzling, continued it even though "no treasury in the world would be equal to the cost of this war," and that "there would not be time or money enough in the world to reduce by force the 24 towns which have rebelled in Holland, if we are to spend as long in reducing each one of them as we have taken over similar ones so far"?[6]

The answer lies in the Spanish court's perception of the broader value and meaning of the Netherlands. The essential reason for Spain's insistence on fighting such a costly war was not the value of the Low Countries themselves, but rather strategic considerations vis-à-vis third parties—in particular France and England, as well as other parts of the Empire. Had only the Netherlands been at stake, Philip would most likely have given up the fight rapidly. Yet he understood that maintaining order in the Netherlands also had implications for Spain's future bargaining power.[7] No peaceful agreement was reached because the pie at stake was broader than the Netherlands alone, so that Spain was not willing to make any concession—they would have had too dire consequences for the Empire.

First, the Spanish leaders believed that the outcome of the war in the Low Countries would have an impact on other parts of the Empire. Ministers argued, for example, that "if the rebellion in the Low Countries is allowed to continue, Lombardy and Naples will follow," and that

> "the first and greatest dangers are those that threaten Lombardy, the Netherlands and Germany. A defeat in any of these three is fatal for this monarchy, so much so that if the defeat in those parts is a great one, the rest of the monarchy will collapse; for Germany will be followed by Italy and the Netherlands, and the Netherlands will be followed by America; and Lombardy will be followed by Naples and Sicily, without the possibility of being able to defend either."[8]

Others contended that even more could be lost: "if we lose Flanders, we will also lose the Indies and other Kingdoms;" "Spain will enjoy neither peace nor security, so that the true and perhaps the only way to secure Spain and also the other states [of his Majesty] is to make the supreme and ultimate effort to cure the Netherlands [problem] now;" and, even more dramatically, "keeping Flanders is so essential that I very much fear that the Spanish Monarchy will not last long if we lose it."[9]

A second concern was reputation. Failing to fight would "strain Your Majesty's conscience and hazard your honor and prestige" and was incompatible "with the honour and reputation of Your Majesty, which is your greatest asset."[10] Philip's concern was also that any sign of weakness in the Netherlands would reduce Spain's prestige and international stature as a world power. "We should consider the issue of religion not only as a matter of piety and spiritual obligation, but also as a temporal one involving reputation."[11]

Third, the war was believed to act as a "punching ball" that focalized other enemies' resources, and hence alleviated the costs and pressures on Spain (Parker 1972):

> "Although the war which we have fought in the Netherlands has exhausted our treasury and forced us into the debts that we have incurred, it has also diverted our enemies in those parts so that, had we not done so, it is certain that we would have had war in Spain or somewhere nearer."[12]

Finally, the conflict in the Low Countries was part of a larger power struggle for hegemony in Europe between the Spanish Empire and France: "the surest means we have of keeping the French in check is to maintain strong forces in Flanders."[13] Abandoning Flanders would mean that France was likely to control it—a dangerous concession to one of Spain's most serious enemies:

> "[The Low Countries] are the bridle which restrains and curbs the French, the English and the rebels, whose forces, should that shield fail, would fall on Your Majesty and his Kingdoms in several parts, giving rise to greater expenses and dangers."[14]

In short, fighting was justified as an investment—one that would avoid further costs and loss of prestige, and hence of bargaining power.

The reasoning of the Spanish king is not an isolated case in history. George the Third, for example, speaking on the necessity of subduing the American colonies in 1779, thought that "it is necessary [...] to weigh whether expences [of war] though very great are not sometimes necessary to prevent what might be more ruinous to a country than the loss of money:"[15]

> "Should America succeed in that, the West Indies must follow them [...]. Ireland would soon follow the same plan and be a separate state; then this island would be reduced to itself, and soon would be a poor island indeed."[16]

The United States' containment doctrine is another example of the incentive to engage in wars of investment. Korea and Vietnam were not defended for their economic or military value, but rather because losing them would "promote recklessness by great powers who have not abandoned goals of world conquest" and "result in a collapse of confidence in American leadership, not only in Asia but throughout the world."[17] Vietnam in particular was a war of investment—a war waged to increase, or at least to avoid losing bargaining power vis-à-vis other nations.[18]

> "Without accepting the pat simplicities of domino theory, none of us can doubt that preservation of the independence of Thailand, of

> Malaysia, of Singapore and Burma—and behind them, in the long run, Indonesia, India, the Philippines and Australia—will become infinitely more difficult if this [communist] venture succeeds in South Vietnam."[19]

Similarly, French politicians and officials in the post-WWII period feared that nationalism would spread rapidly throughout the French Union if they did not curb nationalism in Indochina, and that this would lead to the destruction of the Empire.

1.2 Endogenizing Power

Unfortunately, power is usually taken as exogenous in the literature, which makes it difficult to explain the class of wars that the Dutch revolt illustrates. Fearon (1995), for example, takes probabilities of prevailing in a conflict as a primitive, from which he derives various results about the rationality of war. Such mappings are insufficient, however, as they ignore changes in power driven by the players' strategies themselves, and hence rule out by assumption the possibility to fight over capabilities. Yet, if certain attributes enable players to affect others' utility and hence determine their share of the pie, then these players should value these attributes and behave strategically to acquire them. States, for example, often go to war not only to gain direct benefits, but also to accumulate resources that will increase their future power.

Fearon (1996) analyzes this very problem in the case of two nations bargaining over the division of a pie when present gains affect future bargaining power. He reaches the striking conclusion that war never occurs in equilibrium, as long as the mapping from resources (territory) to power (the likelihood to prevail in a conflict) is continuous. In equilibrium, one of the players essentially makes a demand such that the future expected stream of resources is sufficiently large to satisfy the other player, even if this means that in the long run, the other player's territory will completely vanish.[20] As we show in section 2.1, this result holds even if we relax the assumption that war is an all-or-nothing contest (i.e., victory would mean the conquest of a portion only of the other's territory). However, we also show that the result fails if the two states value the future too differently.

Garfinkel & Skaperdas (2000) explore a slightly different setup in which states allocate resources to either guns—a determinant of their power—or butter—which affects their utility. They find that war can occur in equilibrium if the players value the future sufficiently. The result hinges on the idea that getting rid of the opponent today implies saving on defense costs tomorrow. In other words, even peace in a lawless environment is inefficient, since resources must be expended on defense costs. In a sense, this is equivalent to paying a cost of war in each period. If that cost is sufficiently large (and the players care sufficiently about the future), then paying this cost once and for all early on is rational. The mechanism by which war occurs in the present paper is related, but different. Here, a state fights another not to save on defense costs (or, equivalently, destroy a front and allocate the thereby saved resources to another), but rather as a way to increase its resources, and hence the capabilities it can use in negotiations against a third party.

To be sure, the idea that states might be willing to fight over resources that affect their future power is not new, and is perhaps best expressed in the work of realists such as Morgenthau (1948). Yet, realists have typically *assumed* the profitability of investments in additional sources of power, without necessarily questioning the conditions under which this is the case. Thus, even though it is probably accurate that "the more powerful [states] are relative to their rivals, the better their chances of survival" (Mearsheimer 2001), why states would fail to reach an agreement and fight as a result is not as obvious. In fact, I show that war does not occur even when present gains affect future power, unless these gains increase future power by a factor greater than the increase in capabilities (returns are 'increasing'). Similarly, balance of power theories typically rely on the idea that states are concerned about the accumulation of power and, as a result, will coalesce against "expansionist" states.[21] Yet, the result presented below implies that expansionist states need not be a source of concern for their neighbors unless there are increasing returns.

As we will show, the real question is how much capabilities "afford" the players. Does doubling capabilities more or less than double power? The answer to this question will tell us when fighting to acquire resources that increase future bargaining power is rational, and when expansionist states should be feared. They will give us clearer predictions about when we should expect wars to occur, as opposed to realist explanations which, although they can always justify war ex post as the result of states' pursuit of power, cannot tell us ex ante when war should or should break out.

The ability to extract resources from conquered states has received some attention in the empirical literature (Liberman 1996). However, existing work has focused on rates of extraction, or "cumulativity"—in other words, how conquests affect capabilities ($Resources \longrightarrow Capabilities$). They do not ask whether conquest is worth pursuing, but rather how much can be extracted and how "cumulative" these resources are (Van Evera, 1999). Instead, this paper deals with the value of these additional capabilities ($Capabilities \longrightarrow Power$)—that is, how they affect the conqueror's chance of prevailing in future conflicts—rather than the amount of capabilities states can extract from conquered territories.

Finally, the idea that present outcomes have an impact on future interactions has been analyzed in a number of contexts in biology and economics under the label of "self-reinforcing mechanisms," "positive feedback," or increasing returns (Buchanan & Yoon 1994). In international relations, increasing returns to scale are perhaps most obvious in so-called "domino" effects.[22] The argument is generally that losing a small amount of the pie today leads to even larger losses tomorrow. This is a clear case in which a small decline (increase) in resources (e.g. loss of a small ally during the cold war) can lead to a more than proportional decrease (increase) in power, and hence in benefits. Our goal is here is to derive more formally the conditions under which such situations might cause war.

2 When Do States Invest in War?

2.1 Two-state model

In this section, we show that the counterintuitive result derived in Fearon (1996) is not contingent upon the assumption that war is an all or nothing endeavor. Indeed, while Fearon's model concludes that war never occurs in equilibrium (provided that $p(x)$ is continuous), this might be simply because war terminates the game, and hence players have no incentive to 'invest' in war—to fight now to increase their future power. To rule out this possibility, I therefore present a simple variation on Fearon (1996), and show that war never occurs in equilibrium, even if the outcome of war is a new partition of the pie.

Two players, A and B, negotiate over the partition of a territory X of size

1. The game takes place in two identical stages indexed by $t \in \{1, 2\}$, and both players have complete information throughout the game. In each stage, A proposes a partition of the territory $x_t \in [0, 1]$, where x_t denotes A's share and $1 - x_t$ is B's share. B can then either accept or reject this offer. If the offer is rejected, then a 'war' is fought. The outcome of the war is a new partition $(y, 1 - y)$, where y is the realization of a random variable Y (see Fig. 1 bottom) with continuous probability density function $f(y|x_{t-1})$ (x_0 is the distribution at the beginning of the game). Note how the probability distribution is conditional upon the existing partition of the territory (Fig. 1).

[**FIGURE 1 ABOUT HERE**]

That is, the expected amount of territory gained (or lost) in a military confrontation depends on the current distribution of territory. Regardless of the outcome—war or an agreement—states receive utility $u_i(x_i)$, and the game proceeds to the next stage (or stops at the end of stage 2). To facilitate exposition, we assume that players are risk-neutral (i.e., $u_i(x) = x$). Finally, we assume that both states discount the future at a rate δ and that warring states incur a one-time loss of utility c_i.

Proposition 2.1. *War never occurs in equilibrium.*

The intuition behind this result is that A's total gain over the 2 periods is exactly B's loss, and hence there must be some intertemporal agreement such that the sum of the benefits received by B can remain constant. An *intertemporal* agreement is crucial here, as A might need to grant B more today than she would in a single-stage game, since B needs to be compensated for the expected loss of power tomorrow.

However, intertemporal concessions might not work if the players' discount factors differ too much (Chadefaux 2011). To see this, consider the extreme case in which $\delta_A = 0$ and $\delta_B = 1$ (i.e., A does not value the future at all, and B does not discount it). Then the least A wants for herself is the equivalent of her instantaneous expected return to fighting, $x^* = E[x_1|x_0]$. But if B were to accept, this could mean a much higher level of power for A in the next round, which is not acceptable for B. As a result, B rejects the offer and war occurs. In other words, the problem with different discount factors is that the two players cannot smoothen the benefits over several period. When both players value the future at the same rate, one of them accepts a small concession today because she expects it to be compensated by a higher level of power tomorrow. If, however, that state does not value the future, then the expected compensation tomorrow is worthless, and hence it will demand more today—more than the other state is willing to concede.

Note however that the problem caused by different discount factors is not directly related to the mechanism of interest here—wars of investment. Indeed, the problem here is one of commitment, in the sense that while there exists a deal that both would prefer to war (A would give B more today and less tomorrow), B has no guarantee that B will respect that agreement tomorrow. Because of this inability to commit, war occurs.

2.2 Three-state model

We show now that the results are quite different in the case of a three-player game. Suppose for example that there are three states indexed by $i \in \{A, B, C\}$ and two pies, x_{AB} and x_{AC}, each of size one. Player i's share of pie x_{ij} is denoted by x^i_{ij}. All players have complete information throughout the game. Probabilities of winning a war are defined as $p(\mathbf{x}^i, \mathbf{x}^j)$, where $\mathbf{x}^i = x^i_{ij} + x^i_{ik}$, for $i \neq j \neq k$.[23] That is, the probability of winning the war and capturing territory x_{ij} is a function of the players' total territory at time t. The intuition is that states can extract from these territories the resources necessary to wage war. The game proceeds in two stages, denoted by $t \in \{0, 1\}$.

Stage 1: In the first stage, A and B bargain over $x_{_{AB}}$, of size normalized to one. A proposes a partition $x_{_{AB}}$, which B can accept or reject. If B rejects, then war follows—a lottery in which A wins the entire pie $x_{_{AB}}$ with probability $p^t_{_{AB}} = p(\mathbf{x}^A, \mathbf{x}^B)$.[24] In addition, both incur the costs of war—a one-time loss of utility $c^i_{ij} > 0$.[25] If instead B agrees to the partition, then each player receives the specified share. The game proceeds to stage 2 following either war or an agreement.

Stage 2: In the second stage, A and C bargain or fight following the same protocol (i.e., agree on a partition or fight). Note however that A's capabilities now incorporate her gains or losses of the first stage. Intuitively, A can extract resources from the share of $x_{_{AB}}$ she obtained and transform them into capabilities (e.g., use the occupied territory's manufactures to build military material).[26] Finally, we assume again for clarity that the players are risk neutral ($u_i(x) = x$).

When Does War Occur?

Does war occur in equilibrium and, if it does, under what conditions? First, note that war never occurs in the last stage of the game, since it is strategically equivalent to Fearon (1995)'s setup.

Lemma 2.1. *War never occurs in stage 2.*

Consider now A's incentives when facing B in the first stage. A knows that the outcome of this negotiation matters not only for what she obtains now (her share of x_{AB}), but also for her ability to bargain with C in the next stage (her share of x_{AC}). Hence, A might be willing to incur significant costs to acquire additional capabilities if these capabilities increase her bargaining power versus C. This implies that even the largest concession that B is willing to make might be insufficient to satisfy A. Indeed, I show in proposition 2.2 that, when present gains affect future power, two states can have a positive expected utility for war in the dyad, even without risk-acceptance or asymmetric information.

To make this clearer, let p^H_{AC} denote the probability that A will prevail in a fight against C after defeating B, and p^ω_{AC} denote that same probability after reaching the best possible agreement with B.

Proposition 2.2. *War occurs in equilibrium in stage 1 if*

$$(p^H_{AC} - p^L_{AC})p_{AB} > (p^\omega_{AC} - p^L_{AC}) + \frac{c^B_{AB} + c^A_{AB}}{x_{AC}}$$

In short, condition 2.2 shows that war occurs if the added capabilities gained by defeating B, weighted by the probability that A actually wins (LHS), increases A's chance of winning against C more than any agreement that B is willing to sign (RHS). When this is the case, the most B is willing to concede remains insufficient to satisfy A (see appendix A.1 for an example)

More generally, the result that war does not occur with complete information fails when there exist situations in which half the pie is not half as good as the entire pie. This is well recognized by Fearon in the case of risk-acceptance, for example. In the present model, however, such increasing returns emerge because present gains affect future power. Controlling the entire AB pie can be more than twice better than only half of it, because it increases the likelihood of prevailing against C more than proportionally. As a result, countries may undertake what appears as risk-acceptant actions at the dyadic level in order to reap larger benefits at the global level.[27]

The most important difference with the two-player bargaining game is that here, A and B are indirectly negotiating over C—a territory over which neither has control. This means that A's gain is not exactly B's loss, and B cannot concede what A wants, because B does not control it. A is asking for concessions beyond what B can expect from a war, because these concessions have value beyond their negotiation. Moreover, if A does not conquer B to face C with more power, then C might attack B herself to face A favorably (although this aspect is not included in the simple model above).

An important corollary of proposition 2.2 is that war occurs *only if* the returns to additional capabilities are increasing. That is, A has an incentive to fight only if an additional unit of territory increases her likelihood of winning against C more than proportionally. Practically, it means that war occurs only in situations in which obtaining half the initial pie is not half as useful as obtaining it all.

Proposition 2.3. *War occurs in equilibrium in stage 1 only if* $p_{AC}(\mathbf{x}^A, \mathbf{x}^C)$ *is convex in* $\mathbf{x}^A$ *on the interval* $\left[x^A_{AC}, x^A_{AC} + 1\right]$.[28]

This is counterintuitive. It implies that, no matter how much an additional unit of capabilities increases A's likelihood of prevailing against C, A has no incentive to fight B to acquire this extra unit as long as p_{AC} is linear or concave—that is, as long as the marginal returns to an additional unit of capability are non-increasing. Thus, the result holds for example for *any* β in $p_{ij} = \frac{\beta r_i}{\beta r_i + r_j}$, a typical contest success function that is not convex but that can be very steep for high values of β (Hirshleifer 2001).

The logic is the following. If doubling capabilities does not more than double power, then half the amount of capabilities is at least half as good as all the capabilities. An agreement at $x^A_{AB} = 0.5$, then, is at least as good as a .5 probability chance of winning $1x_{AB}$ by fighting. If, on the other hand, doubling capabilities increases future power by a factor greater than two, then fighting can be preferable. If, for example, obtaining $1x_{AB}$ leads to gains against C more than twice higher than those that would have been obtained with an agreement at $\frac{1}{2}x_{AB}$, then fighting now can be a best response for both parties, since A will not be satisfied with any concessions that B is willing to make.

At a more general level, the problem with increasing returns is that they generate situations that are strategically equivalent to ones of indivisibilities or of risk-acceptance, in the sense that a partition of the pie provides a lower total utility than the whole pie, so that there exists no self-enforcing equilibrium that both parties in a dyad prefer to war.[29] Thus, A rejects B's offer to split the AB pie in, say, half, because securing the whole pie more than doubles A's final utility. The reason war occurs is not A valuation of the AB pie itself, but rather because of what its control implies for her ability to extract benefits from C.

Proposition 2.3 has important implications for the realist paradigm. It demonstrates that war arises in the first stage only when the returns to additional capabilities are increasing. States will fight to acquire power only if the expected increase in capabilities from fighting B increases their likelihood of prevailing in future encounters more than proportionally—that is, if the marginal gain in power derived from an additional unit of capability is increasing. States can be expansionist, but only when the mapping from capabilities to power is convex. This gives us practical predictions about the occurrence of war. Under complete information, war only occurs in the presence of increasing returns—or more precisely, when the players *perceive* returns to be increasing. The drive to fight to acquire power is not as universal through space and time as we might have expected.

Do Alliances Prevent War?

In the previous sections, third parties were modeled as passive—C did not participate until the second round. This was useful to isolate A's incentives but, in the real world, third parties need not stay inactive. In particular, how does the possibility to form alliances affect the results obtained so far? While limited space prevents us from examining a complete model of alliances, I sketch the logic of a model to show when alliances form and prevent war—and when they do not.

Consider first the case of defensive alliances—by far the most prevalent form (Walt 1987, Leeds 2003). They are agreements by which C offers B support in case of an aggression by A. Defensive alliances will avoid war only if at least two conditions are met. First, C needs to have an incentive to ally with B. Defending an ally is risky and costly, and so C might prefer abandoning B. In other words, defending B must be in C's interest if the alliance is to be credible. Second, an alliance needs to be sufficiently powerful to deter A from attacking B regardless.

To model defensive alliances, suppose that C has the choice of offering B some support—a "subsidy" of size s, which is paid if A attacks B, and which has a cost (in utility) of $f(s)$. Let p^{s}_{AB} and $p^{\not{s}}_{AB}$ denote A's likelihood of winning against a B who receives C's support, and one who does not, respectively.[30]

Lemma 2.2. *C forms a defensive alliance with B only if*

$$f(s) < \left(p^{S}_{AB} - p^{s}_{AB}\right)\left(p^{H}_{AC} + p^{L}_{AC}\right).$$

Three main factors affect C's choice to support B or not. First, for sufficiently large values of f, C will prefer not supporting B. A large f could be due, for example, to the difficulty or financial cost of moving troops, the geographical distance or the audience costs incurred (Morrow 1993). Second, the efficiency of C's support—how it affects A's likelihood of prevailing against B—is also determinant. In particular, the larger the effect of the subsidy (the larger $(p^{S}_{AB} - p^{S}_{AB})$), the higher a price C is willing to pay to support B. Finally, the extent of the difference between facing a strong A (one that has defeated B) and a weak one (one that lost against B) also affects C's calculus. As the difference between facing a victorious A and a defeated A decreases (i.e., as $(p^{H}_{AC} - p^{L}_{AC})$ decreases), so does C's willingness to support B. Beyond these particular results, what matters is that the possibility to form defensive alliances is not always sufficient to prevent war, even without asymmetric information or miscalculation. If the cost for C of allying with B is too high, or if the returns to its support are low, then letting A attack B is a best response.

Consider now offensive alliances—situations in which A allies with B against C to extract a potentially larger share of x_{AC}, without incurring the costs of an initial war of investment against B. While a complete model of offensive alliances is well beyond the scope of this paper, I elaborate on a number of reasons why such alliances might fail to form, and why war might still occur. First, note that alliances can be inefficient mechanisms of capability aggregation, such that the likelihood of winning of an alliance can be lower than if the resources had been pooled and used under a single country's banner (Morrow 2000). Second, an alliance presents significant risks for A. B might promise to fight alongside A in case of a war with C, but end up being less committed than initially agreed upon, or even defecting to C. Finally, supporting A is dangerous for B. Indeed, once A has extracted more resources from C thanks to her alliance with B, there is no guarantee for B that A will not use this additional power to exploit him. "To ally with the dominant power means placing one's trust in its continued benevolence" (Walt 1985).

3 When Are Investments Profitable?

Fighting is a way to acquire resources that increase future power—and hence the probability to survive and/or expand. War, however, only occurs in situations in which doubling capabilities more than doubles power. Given the importance of these increasing returns to the mechanism presented above—and hence to our understanding of the causes of war—I discuss in this section some situations in which they are likely to emerge. I then examine factors that reduce the returns to additional capabilities—in other words, reasons why incentives to acquire additional capabilities can be limited.

3.1 Sources of Increasing Returns

What factors affect returns on investment? More specifically, when is the marginal effect of additional capabilities on power increasing? In this section, I explore a number of conditions under which additional capabilities significantly increase the likelihood of securing a favorable outcome at the negotiation table, and hence in which wars of investment are most likely to occur.

First, an additional capability can increase the likelihood of winning in a military conflict more than proportionally (Hirshleifer 2001). Thus, Lanchester's Square Law, widely used in official combat models used by the Defense Department, states that the power of an army in modern combat is proportional not to its number of units, but to the square of this number, and hence is convex (Lanchester 1956). To be sure, empirical evidence for the theory is mixed (Biddle 2004). However, the claim here is not that a large N study of battles need verify the empirical validity of the laws. Rather, it is that there exist situations in which the laws apply, and these situations are the ones in which incentives to fight exist.

Economies of scale are another mechanism leading to increasing returns, since they lead to situations in which multiplying inputs by two (say, territory) increases output (say, GDP) by more than two. Sources of economies of scale are multiple, and I mention only a few examples here. A large market, for example, generates clear economies of scale. In per capita terms, it is cheaper to "buy" defense in a larger country. Similarly, the ratio of government spending over GDP should be larger in smaller countries (Alesina & Spolaore 2003).

In addition, there exist thresholds of capabilities at which players are able to reap large benefits. Mearsheimer argues for example that states who control more than half the resources cannot anymore be deterred by balance of power mechanisms. Similarly, there values of voting rights that provide the player with veto power, for example, or a sufficient critical mass. Thus, voting rights in international organizations (e.g. EU, IMF,...) are often determined on the basis of the player's capabilities (population, contributions, GDP), so that being just a little bigger can make the difference between being in or out (e.g., the "Green room" at the WTO, or the UN Security Council). Finally, conquering 100% of an opponent's territory is qualitatively different from conquering only 99% of it. Garfinkel and Skaperdas, for example, show that war can rationally occur because of the savings in defense spending such conquest implies. Eliminating an opponent now saves spending on military defense in the future. This is a case where controlling 99% of the pie is fundamentally different from controlling 100%, and hence in which increasing returns are the underlying cause of war.[31]

3.2 Limits to Increasing Returns

I explore in this section a number of mechanisms that limit the incentive to fight to acquire capabilities. In essence, these mechanisms can transform a convex function (one with increasing returns) such as the mapping from resources (territory) to the likelihood of prevailing, to a concave one, and hence compensate potential increasing returns in the mapping from capabilities to power.

Frictions

'Frictions' in one of the mappings from resources to utility can affect the magnitude of returns. For example, any incentive to challenge the status quo is always limited by the size of the initiator's cost of war. If war is expected to be costly, any threat to change the status quo by force to reflect a slight change in relative power is not credible. In other words, returns to capabilities need to be very high if the cost of war is high, as is clear from proposition 2.2.

Another source of friction is the level of cumulativity. Although this paper mostly addressed the mapping from capabilities to power, the mapping from resources to capabilities also matters. It is explored, for example, in Van Evera (1999) and Liberman (1996). Resource cumulativity refers to the ability to transform additional resources into actual capabilities. The level of extraction, for example, might be low, such that an additional piece of territory or a greater population do not change the level of capabilities much.

Finally, risk aversion—more precisely, the concavity of $u_i(\cdot)$ can also limit incentives to fight. If the convexity of $p(\cdot)$ is more than compensated by the concavity of $u(\cdot)$, then the overall mapping from resources to utility does not exhibit increasing returns, and hence war is not rational. In this case, the value of any gamble is discounted compared to the value of the certain outcome. As a result, even if the mapping from capabilities to power exhibits increasing returns, concavity in $u(\cdot)$ can make the overall mapping from resources to utility concave.

Immediate Costs, Long-term Rewards

The incentive to invest is further limited in a more subtle fashion by the risk that the aggressor incurs vis-à-vis third parties. To attack B, A must expand scarce resources and capabilities on one front. Fighting involves short-term costs that change the shape of the mapping from investments to power (figure 2). Although the long-term gains of a victory might easily offset this cost, the loss puts the state at risk vis-à-vis third parties, who might take advantage of this temporary weakness and demand more of him while he is weak.[32] In other words, A's war with B temporarily decreases her ability to fight C, and this creates a window of opportunity during which C can demand more from A.

[FIGURE 2 ABOUT HERE]

In other words, the expanse of resources toward B cannot always be analyzed in an atemporal fashion. Resources are used over time, and such an expansion can put a state at risk in the short run. Fighting involves short-term costs that change the shape of the returns to an initial investment. It generates a temporary weakness from which the conqueror might never recover, and hence the inefficiencies caused by increasing returns are offset by a sufficiently large initial loss.

4 Concluding Remarks

Anarchy alone is insufficient to explain conflict, as Fearon (1995) explains.[33] Instead, one reason why anarchy can become a Hobbesian world is the existence of increasing returns. What makes neighbors threatening is not the concern that they might attack, but rather that they have an incentive to do so, which is the case in the presence of increasing marginal returns. Not taking risks early on can endanger the state, so that war is an investment which can be necessary to the state's survival.[34]

Although the more classical literature has long recognized that war can be an investment for longer-term purposes, it has not derived the specific conditions under which we should expect this to be the case. As a result, both the occurrence of war and its absence can be justified under the banner of the pursuit of power. Understanding the conditions under which such an investment is rational requires that we analyze how accumulated capabilities transform into actual power—the ability to secure a favorable agreement. On the other hand, formal modelers have largely focused on bargaining as a process by which players negotiate over the partition of a pie that is valued for its own sake only. However, few have analyzed the incentive structure that emerges when players can prepare this negotiation stage. As a result, they have largely ignored the possibility that war can be only a part of a broader strategy and, as we saw in the case of the eighty years war, can offer an odd reading of a number of wars.

Overall, the present paper brings together the more classical literature on the causes of war and more recent formal work. It provides micro-foundations for offensive realism, while at the same time extending existing formal work by analyzing the conditions under which negotiations between two and three states can break down into war. Differences in results between realist approaches and bargaining theories can be explained by different assumptions about the mapping from capabilities to power. Increasing returns should be central in realist theories, and they also lead us to reconsider the results obtained in the bargaining literature that both countries cannot have a positive expected utility for war when information is complete. The two strands of the literature examine different parts of a larger game. Bargaining models have limited their attention to the mapping from power to final outcomes and hence have not given players the ability to adopt strategic moves to improve the physical foundation of their long-term power; more general theories have been somewhat vague on their micro-foundations, and have not told us enough about the sufficient or necessary conditions for war. This paper is in part an attempt at reconciling these two approaches.

Appendix

A Examples

A.1. To illustrate how war can occur in equilibrium with three players, suppose that $x^A_{AB} = x^A_{AC} = 0.1$. That is, $\mathbf{x}^A = 0.2$, $\mathbf{x}^B = 0.9$ and $\mathbf{x}^C = 0.9$. For simplicity, suppose also that $c^i_{ij} = 0$ for all $i, j \in \{A, B, C\}$. Finally, assume that $p_{ij} = \frac{(\mathbf{x}^i)^m}{(\mathbf{x}^i)^m + (\mathbf{x}^j)^m}$, with $m = 2$ for this example (see Fig. B).

In stage 1, note that B could always reject A's offer and fight with expected utility $\frac{0.9^2}{0.9^2+0.2^2} \approx 0.9529$. This means that if war is to be avoided, B must receive at least 0.9529, and hence A obtains at most $1 - 0.9529 = 0.047$. Following this agreement, C would then demand at least $\frac{0.9^2}{0.9^2+(0.1+0.047)^2} \approx 0.9973$ in the second round, so that A would receive 0.0027. So in total, the best A can hope from an agreement is $0.047 + 0.0027 = 0.0498$.

Consider now A's expected utility for fighting B. With probability 0.047, A wins the whole AB pie and goes on to stage 2, where C demands at least $\frac{0.9^2}{0.9^2+(1+0.1)^2} \approx 0.4$, leaving A with 0.6. But with probability 1-0.047, A loses the whole AB pie and goes on to stage 2, where C demands at least $\frac{0.9^2}{0.9^2+(0+0.1)^2} \approx 0.9878$, leaving A with 0.0122. So the expected value of fighting is

$$0.047 \times (1 + 0.6) + (1 - 0.047) \times (0 + 0.0122) \approx 0.087$$

But note that this is higher than the utility derived from the best possible agreement, and hence that A prefers war to any agreement B is willing to accept.

B Proofs

Proof of proposition 2.1. Consider first stage 2. Here, B can always reject A's offer and fight, with expected utility

$$\text{A:} \quad \int_0^1 f(x_2|x_1)x_2 \mathrm{d}x_2 - c_A = E[x_2|x_1] - c_A$$

$$\text{B:} \quad \int_0^1 f(x_2|x_1)(1-x_2)\mathrm{d}x_2 - c_B = 1 - E[x_2|x_1] - c_B$$

This means that if A is to avoid war, it can at most offer x such that $u_B(x) = 1 - E[x_2|x_1] - c_B$. Consider now stage one. Here, players must consider the impact of the distribution of territory at the end of this round on the next stage's equilibrium. Thus, rejecting A's offer x^* (i.e., fighting) would yield an expected utility of

$$1 - E[x_1|x_0] - c_B + \delta \int_0^1 E[x_2|f(x_1|x_0)] - c_B \mathrm{d}x_1. \tag{1}$$

and is defined similarly for A (replacing $1 - E[\cdot]$ with $E[\cdot]$, and the second $-c_B$ with $+c_B$).[35] The first part is the expected distribution of territory resulting from war in stage 2, whereas in the second part, every expected distribution is weighted by the probability of that specific distribution being the outcome of stage 1. So if war is to be avoided, there must exist x^* such that

$$x^*+\delta(E[x_2|x^*]+c_B)\geq E[x_1|x_0]-c_A+\delta\int_0^1 E[x_2|f(x_1|x_0)]+c_B\mathrm{d}x_1. \tag{2}$$

and

$$x^*+\delta(E[x_2|x^*]+c_B)\leq E[x_1|x_0]+c_B+\delta\int_0^1 (E[x_2|f(x_1|x_0)])+c_B\mathrm{d}x_1\ .^1 \tag{3}$$

Since $c_i\geq 0$, both conditions will be satisfied if there exists an x^* such that:

$$x^*+\delta(E[x_2|x^*])=E[x_1|x_0]+\delta\int_0^1 (E[x_2|f(x_1|x_0)])\mathrm{d}x_1. \tag{4}$$

By continuity of $f(\cdot)$, it is easy to see that there must exist an x^* that satisfies (4). From (2) and (3), it is also clear that accepting x^* dominates rejecting it for B, and similarly for A that proposing x^* dominates proposing x' such that B would reject x'. Therefore, war never occurs in equilibrium. □

Proof of lemma 2.1. In the second stage, the setup is the same as Fearon (1995). In short, because information is complete and war is costly, an agreement is reached immediately and war never occurs. □

Proof of Proposition 2.2. First note that B will reject any agreement that yields

[1]This comes from simply rewriting $1-x^*+\delta(1-E[x_2|x^*]-c_B)\geq 1-E[x_1|x_0]-c_B+\delta\int_0^1(1-E[x_2|f(x_1|x_0)])-c_B\mathrm{d}x_1$.

a utility lower than fighting—that is, any agreement such that:

$$1 - x^B_{AB} < (0)p_{AB} + (1 - p_{AB})x_{AB} - c^B_{AB}$$
$$< 1 - p_{AB} - c^B_{AB}$$

This means that the most A can obtain in an agreement is $p_{AB} + c^B_{AB}$. Similarly, in stage 2, A can demand at most $p_{AC} + c^C_{AC}$ from C. This means that without war, A can hope for at most

$$p_{AB} + c^B_{AB} + p^\omega_{AC} + c^C_{AC}, \tag{5}$$

where $p^\omega_{AC} = p(x^A_{AB} + x^A_{AC}, \mathbf{x}^C)$. Consider now A's utility from fighting B. Let $p^H_{AC} = p(x_{AB} + x^A_{AC}, \mathbf{x}^C)$ and $p^L_{AC} = p(x_{AB}, \mathbf{x}^C)$. Then A's expected utility for fighting B in stage 1 is:

$$(1 + p^H_{AC})p_{AB} + (0 + p^L_{AC})(1 - p_{AB}) - c^A_{AB} \tag{6}$$

Then A will fight if (6) > (5), that is if

$$(p^H_{AC} - p^L_{AC})p_{AB} > (p^\omega_{AC} - p^L_{AC}) + \frac{c^B_{AB} + c^A_{AB}}{x_{AC}}$$

□

Proof of Proposition 2.3. First note that A's utility for fighting B is given by:

$$\left[1 + p(x^A_{AC} + x_{AB}, \mathbf{x}^C)\right] p_{AB} + \left[0 + p(x^A_{AC}, \mathbf{x}^C)\right] (1 - p_{AB}) - c^A_{AB} \tag{7}$$

and her utility for the most B is willing to concede is:

$$\begin{aligned} & p_{AB} + c_{AB}{}^{B} + p_{AC}\left(x^A_{AC} + p_{AB} + c_{AB}, \mathbf{x}^C\right) \\ & \geq p_{AB} + c^B_{AB} + p_{AC}(x^A_{AC} + p_{AB}, \mathbf{x}^C) \quad \text{since } c_i > 0, \end{aligned} \tag{8}$$

So war will occur only if (7) > (8), that is if (after rearranging and simplifying):

$$(1 - p_{AB})p_{AC}(x^A_{AC}, \mathbf{x}^C) + p_{AC}(x^A_{AC} + 1, \mathbf{x}^C)p_{AB} > p_{AC}(x^A_{AC} + p_{AB}, \mathbf{x}^C) \tag{9}$$

Recall now that a function f is convex on an interval I if, for any two points a and b in I and any $t \in [0, 1]$,

$$(1 - t)f(a) + tf(b) \geq f\left((1 - t)a + tb\right)$$

for all $a, b \in I$ and all $t \in [0, 1]$. But note that the RHS of 9 can be rewritten as:

$$p_{AC}(x^A_{AC} + p_{AB}, \mathbf{x}^C) = p_{AC}\left((1 - p_{AB})x^A_{AC} + p_{AB}(x^A_{AC} + 1), \mathbf{x}^C\right),$$

which means that (9) holds only if $p_{AC}(\mathbf{x}^A, \mathbf{x}^C)$ is convex in $\mathbf{x}^A$. □

Proof of Lemma 2.2. C will support B only if the net benefits of her support in case of an attack by A against B are greater than those of not supporting B. That is, C will support B only if:

$$\begin{aligned} & p^s_{AB}p^L_{AC} + (1 - p^s_{AB})p^H_{AC} - f(s) > p^{\mathrm{s}}_{AB}p^L_{AC} + (1 - p^{\mathrm{s}}_{AB})p^H_{AC} \\ & (p^s_{AB} - p^{\mathrm{s}}_{AB})(p^L_{AC} + p^H_{AC}) > f(s) \end{aligned}$$

□

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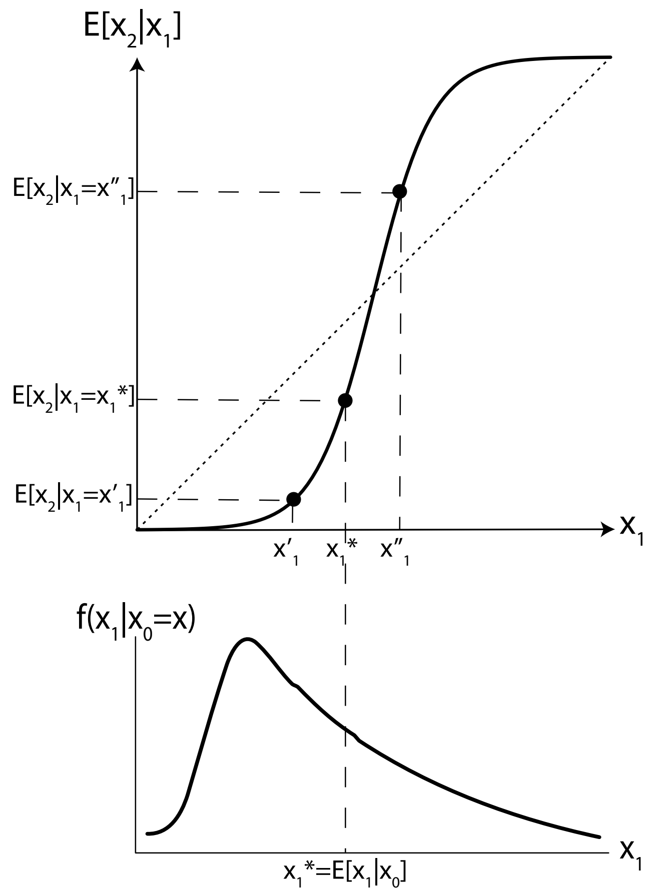

Figure 1: Illustration of the two-player model with a sample mapping from today's distribution to tomorrow's expected distribution of territory. Suppose that A and B agree upon x_1*. Then A's expected share in stage 2 will be $E[x_2|x_1 = x_1*] = \int_0^1 f(x_2|x_1 = x_1*)$.

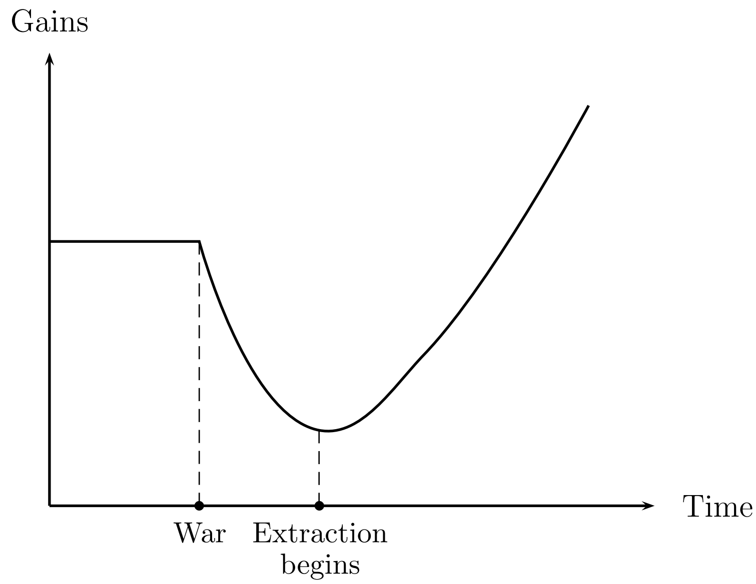

Figure 2: Short term losses, long term gains

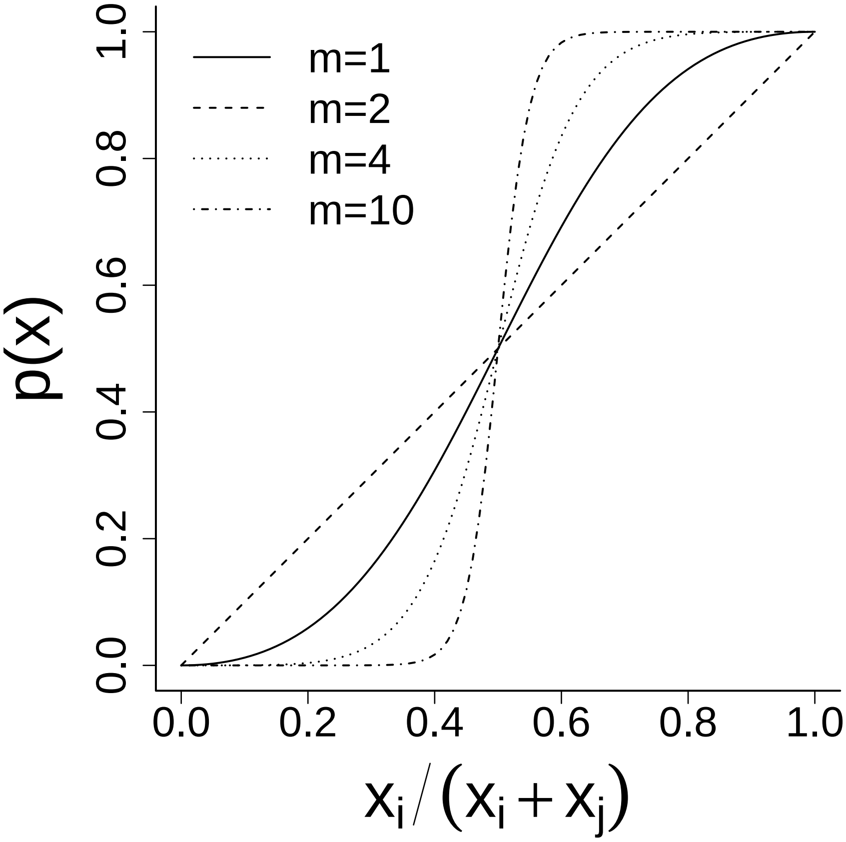

Figure 3: Mappings from resources (x_i) to likelihood of winning.

Notes

[1]For a review of bargaining theory and international conflict, see Powell (2002).

[2]Powell (2006) shows that indivisibilities a a special case of commitment problems, broadly construed.

[3]For an estimation of the war's cost, see Parker (1975).

[4]Instituto de Valencia de Don Juan, envio 51 fo 31, Mateo Vazquez to the king with holograph royal reply, 31 May 1574, quoted in Parker (1976). Council of State, Ibid.

[5]Even the possibility that Philip II was not "rational"—and hence that existing rationalist explanations do not apply—is doubtful, for even a critic admitted that "His Majesty's brain must be the largest in the world".

[6] Archivo General de Simancas, Estado 560 fo 33, Don Luis de Requesens (Spanish commander-in-chief) to the king, 7 November 1574, quoted in Parker (1975). Nueva Colección de Documentos Inéditos para la historia de España, v (Madrid, 1894), p. 368, Requesens to the king, 6 October 1574, quoted in Parker (1975).

[7]For an overview of how the Netherlands fit into Philip II's grand strategy, see Gonzáles de León & Parker (2001).

[8]Report of d'Assonleville, Dec. 1558, quoted in Parker (1998, p. 90). These beliefs were not entirely unfounded, as rebels in Naples cited the Dutch example as an inspiration (Parker 1972). The quote is from an advisor to the Spanish Emperor

(Kennedy 1987, p. 51).

[9]Advisor to the Council of State, 19 Oct. 1629; Marquis of Aytona to Olivares, 29 Dec. 1633; Cardinal Alexandrino to Nuncio Castagna; Aviso of Juan Andrea Doria, 1605. See Parker (1972, p. 111) and Parker (1998, p. 90) for these and further quotes.

[10]Philip II to Alba, 1573, quoted in Parker (1998, p. 89).

[11]Olivares to the King in 1628, quoted in Parker (1972).

[12]Quoted in Kennedy (1987, p.51).

[13]Cardinal Granvelle, 1582, quoted in Parker (1972).

[14]Consulta of the Council of State, 21 March 1600, Real Academia de la Historia (1929–34, III, p. 7).

[15]George III's letter to Lord North, June 11, 1779, quoted in Fortescue (1927, Vol. IV, pp. 350–1).

[16]Quoted in Robinson (1906).

[17]Statement by President Nixon on Nov. 3, 1969. Along the same lines, "Around the globe, from Berlin to Thailand, are people whose well-being rest, in part, on the belief that they can count on us if they are attacked. To leave Vietnam to its fate would shake the confidence of all these people in the value of American commitment, the value of America's world. The result would be increased unrest and instability, and even wider war." Speech by Lyndon Johnson at John Hopkins University, April 7, 1965.

[18]For a discussion of domino theory in the context of the cold war, see Jervis & Snyder (1991).

[19]William Bundy, Assistant Secretary of State for Far Eastern Affairs, May 22, 1966, quoted in Girling (1970).

[20]Note however that this result does not apply if we relax the assumption of equal discount factors (Chadefaux 2011)

[21]"In such a system, the growth in any one states power will be eventually checked by a countervailing coalition of others who become fearful of its expansion and the eventual threat it will pose to the system as it makes its bid for hegemony". B. Slantchev, "Territory and Commitment: the Concert of Europe as a Self-Enforcing Mechanism." Manuscript

[22]"Finally, you have broader considerations that might follow what you would call the 'falling domino' principle. You have a row of dominoes set up, you knock over the first one, and what will happen to the last one is the certainty that it will go over very quickly. So you could have a beginning of a disintegration that would have the most profound influences." Dwight D. Eisenhower at a News Conference on April 7, 1954. See *Public Papers of the Presidents: Dwight Eisenhower, 1954*, Washington, DC: U.S. Government Printing Office, 1960, pp. 382–83.

[23]Note that since there are only two territories here, $\mathbf{x}^B = x^B_{AB}$ and $\mathbf{x}^C = x^C_{AC}$.

[24]Hence, B wins with probability $1 - p^t_{AB}$. I assume that $p.(\cdot)$ can take values in the interval $[0, 1]$ and is twice differentiable. Moreover, I assume that $\partial p(\cdot, \cdot)/\partial \mathbf{x}^i \geq 0$.

[25]In the literature, the cost of war has been modeled alternatively as a one-time loss in utility (Fearon 1996) or as a cost that the players incur forever, even after the conflict has ended (Powell 1999). The choice has no qualitative effect on the results here.

[26]See Liberman (1996) for a discussion of the ability to extract resources in various regimes.

[27]More precisely, it is the perception of increasing returns by A that matters. Since information is complete here, the nuance is irrelevant.

[28]Here x^A_{AC} denotes A's share of x_{AC} at the beginning of the game

[29]For a discussion of indivisibilities as a cause of war, see Fearon (1995), Toft (2003) and Goddard (2006).

[30]That is, $p^s_{AB} = p_{AB}({r_0}_A, r_B + s)$ and $p^{\not s}_{AB} = p_{AB}(r^0_A, r_B)$.

[31]Note that these threshold situations are discontinuities in the mapping from resources to power, to which Fearon (1996) had already drawn attention. However, the reasoning here is different. In Fearon's model, the problem was that these discontinuities might prevent the kind of precise intertemporal deals that were necessary to peace. Here the logic is that these jumps in power make it more profitable for A to attack B—to fight a war of investment.

[32]Blainey (1988) makes a related point.

[33]See also Stephen Walt: "Anarchy is a permissive condition rather than an independent causal force," in Walt (2002, p. 211).

[34]"If others, that otherwise would be glad to be at ease within modest bounds,

should not by invasion increase their power, they would not be able, long time, by standing only on their defence, to subsist."

[35]Since this is essentially an ultimatum game, A can offer B x_2 such that 1) holds with equality, and hence capture B's cost of war